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On the propagation in a moving compressible plasma*

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The paper deals with the propagation of electromagnetic waves through a moving compressible plasma Starting from the fundamental equations, potential equations for such a case have been derived. The dispersion relation has also been derived

1. Introduction

The problem of radiation through moving plasma has been studied by Lee & Papas (1965) and Tai (1965) but their formulation has been confined to the plasma of homogeneous and isotropic type In this paper we have considered the moving plasma as compressible but isotropic in nature and we have derived potential equation for such a case The dispersion relation will also be derived.

2. Wave Equations

We consider the observation system K and another system K' moving with constant velocity $\boldsymbol{v}_0$ Then for K' system the Maxwell equation, hydrodynamic continuous equation and equation of motion are given by (Cohen) 1961)

$$\nabla' \times \boldsymbol{E}' = -\frac{\partial}{\partial t'}(\mu_0 \boldsymbol{H}) \qquad \ldots \quad (1a)$$

$$\nabla' \times \boldsymbol{H}' = \frac{\partial}{\partial t'}(\epsilon_0 \boldsymbol{E}') - en_0\boldsymbol{v}' + \boldsymbol{J}'_s \qquad \ldots \quad (1b)$$

$$\nabla' (\mu_0 \boldsymbol{H}') = 0 \qquad \ldots \quad (1c)$$

$$\nabla (\epsilon_0 \boldsymbol{E}') = P'_s \qquad \ldots \quad (1d)$$

$$n_0' \nabla' \cdot \boldsymbol{v}' + \frac{\partial}{\partial t'} n' = 0 \qquad \ldots \quad (2)$$

$$n_0' m' \frac{\partial}{\partial t} v' = -n_0' e \boldsymbol{E}' - m' v'_T{}^2 \nabla' n' \qquad \ldots \quad (3)$$

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where J_s' and P_s' denote the current density of the source and the charge density respectively. n_0' is the average electron density and n' indicates the small deviation from the average electron density. v' denotes electron particle velocity, m' and v' denote the mass and thermal velocity of electrons in K' system

From eqs. (1) to (3) electromagnetic wave E', H' can be represented by eqs. (5) and (6) using the fourth dimensional vector potential $(A', j\phi'/c)$ given by eq (4), namely

$$\mu_0 H' = \nabla' \times A', \quad E' = -\frac{\partial}{\partial t'} A' - \nabla' \phi' \qquad \cdots \quad (4)$$

$$L' A' = -\mu_0 J_s', \qquad \cdots \quad (5a)$$

$$\left(\frac{\partial^2}{\partial t'^2} + \omega_p^2 \right) L_c' L_u' \phi' = \left(\omega_p^2 L_c' - \frac{\partial^2}{\partial t'^2} L_u' \right) \frac{P_s'}{\epsilon_0}, \qquad \cdots \quad (5b)$$

where

$$L_c' = \nabla'^2 - \frac{1}{c^2} \left(\frac{\partial^2}{\partial t'^2} + \omega_p^2 \right), \qquad \cdots \quad (6a)$$

$$L_u' = \nabla'^2 - \frac{1}{v'^2} \left(\frac{\partial^2}{\partial t'^2} + \omega_p^2 \right). \qquad \cdots \quad (6b)$$

Also, ω_p is the plasma frequency being given by

$$\left(\frac{n_0' e^2}{\epsilon_0 m'} \right)^{\frac{1}{2}} = \left(\frac{n_0 e^2}{\epsilon_0 m} \right)^{\frac{1}{2}}$$

transformation system; u_T' is the thermal velocity in K' system and v_T is in the K system.

Electric field E and the flux density B in K system obtained from Maxwell equations are indicated by

$$B = \nabla \times A, \quad E = -\frac{\partial}{\partial t} A - \nabla \phi, \qquad \cdots \quad (7)$$

similar to eq. (4) using the fourth dimensional vector potential. The vector potential A and A' are related to each other by

$$A' = A - \gamma \frac{\phi}{c^2} v_0 + (\gamma - 1) \frac{A \cdot v_c}{v_0^2} v_0. \qquad \cdots \quad (8a)$$

$$\phi' = \gamma(\phi - (A\, v_0)), \qquad \cdots \quad (8b)$$

where

$$\gamma = \frac{1}{\sqrt{1 - v_0^2/c^2}}.$$

Also $\boldsymbol{J}_s'$ and $\boldsymbol{J}_s$ can be represented by the four dimensional vector potential and are related to each other by

$$\boldsymbol{J}_s' = J_s - \gamma \boldsymbol{v}_0 P_s + (\gamma-1) J \frac{\boldsymbol{J}_s \cdot \boldsymbol{v}_0}{v_t^2} \boldsymbol{v}_0, \quad \ldots (9a)$$

$$P_s' = \gamma \left(P_s - \frac{\boldsymbol{J}_s \cdot \boldsymbol{v}_0}{c^2} \right). \quad \ldots (9b)$$

Substituting eqs. (8) and (9) into eq. (5) and after rearranging we get

$$O_p' L_c' L_u' \boldsymbol{A} = -\mu_0 O_p' L_u' \boldsymbol{J}_s + \frac{\boldsymbol{u}_0}{c^2} \gamma \omega_p^2 (L_c' + L_v')(P_s'/\epsilon_0) \quad \ldots (10a)$$

$$O_p' L_c' L_v' \phi = \gamma \omega_p^2 L_c' (P_s'/\epsilon_0) - \gamma L_v' \left[O_p' (\mu_0 \boldsymbol{v}_0 \cdot \boldsymbol{J}_s') + \frac{\partial^2}{\partial t'^2} \left(\frac{P_s'}{\epsilon_0} \right) \right], \quad \ldots (10b)$$

where

$$O_p' = \frac{\partial^2}{\partial t'^2} + \omega_p^2.$$

Now using the Lorentz transformation equation, differential operators L_c', L_u' O_p' are written in terms of L_c, L_v and O_p as

$$L_c = \nabla^2 - \frac{1}{c^2} \left(\frac{\partial^2}{\partial t^2} + \omega_p^2 \right) \quad \ldots 11a)$$

$$L_v = \nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} + \gamma^2 \left(\frac{1}{c^2} - \frac{1}{v_T^2} \right) \left(\frac{\partial}{\partial t} + \boldsymbol{v}_0 \boldsymbol{\nabla}^2 - \frac{\omega^2_p}{v'^2_T} \right), \quad \ldots (11b)$$

$$O_P = \gamma^2 \left(\frac{\partial}{\partial t} + \boldsymbol{v}_0 \cdot \boldsymbol{\nabla} \right)^2 + \omega_p^2. \quad \ldots (11c)$$

Eqs. (11) have been written by using

$$\boldsymbol{\gamma}' = \boldsymbol{\gamma} - \gamma \boldsymbol{v}_0 t + (\gamma-1) \boldsymbol{\gamma} \cdot \frac{\boldsymbol{v}_0}{v_0^2} \boldsymbol{v}_0,$$

$$t' = \gamma \left(t - \boldsymbol{\gamma} \cdot \frac{\boldsymbol{a}_0}{c^2} \right).$$

In eq. (10) the vector potential A and the scalar potential ϕ satisfy the conditions

$$L_c \boldsymbol{A}_1 = -\mu_0 \boldsymbol{J}_s, \quad \ldots (12a)$$

$$O_p L_c A_2 = \gamma^2 \mu_0 \omega_p^2 \left(\boldsymbol{P}_s - \frac{\boldsymbol{J}_s \, \boldsymbol{v}_0}{c^2} \right) \boldsymbol{v}_0, \quad \ldots (12b)$$

$$O_P L_u \boldsymbol{A}_3 = \gamma^2 \mu_0 \omega_p^2 \left(P_s - \frac{\boldsymbol{J}_s \cdot \boldsymbol{v}_0}{c^2} \right) \boldsymbol{v}_0 \quad \ldots (12c)$$

$$O_P L_u \phi_1 = \gamma^2 \omega_p{}^2 \left(P_s - \frac{J_s \cdot v_0}{c^2} \right) \Big/ \epsilon_0, \qquad \dots \quad (12d)$$

$$O_P L_c \phi_2 = -\gamma^2 \left(\frac{\partial}{\partial t} + v_0 \cdot \nabla \right)^2 \left(P_s - \frac{J_s \cdot v_0}{c^2} \right) \Big/ \epsilon_0. \qquad \dots \quad (12e)$$

$$L_c \phi_3 = \gamma^2 \left(\frac{v_0{}^2}{c^2} P_s - \frac{J_s \cdot v_0}{c^2} \right) \Big/ \epsilon_0, \qquad \dots \quad (12f)$$

where

$$\boldsymbol{A} = \boldsymbol{A}_1 + \boldsymbol{A}_2 + \boldsymbol{A}_3, \qquad \dots \quad (13a)$$

$$\phi = \phi_1 + \phi_2 + \phi_3. \qquad \dots \quad (13b)$$

Eqs. (10) to (12) represent the pltential equations for wave propagating through a moving isotropic compressible plasma

3 Dispersion Relation

From eq. (12) it can be seen that in a moving compressible plasma four dimensional vector potential has two kinds of phase velocity components. So putting $\exp(j\omega t - k_v \gamma)$ for L_ω and $\exp(j\omega t - jk_v . \gamma)$ for $\boldsymbol{L}_v$ in eq. (12) we get

$$k_c{}^2 - \frac{1}{c^2}(\omega^2 - \omega_p{}^2) = 0, \qquad \dots \quad (14a)$$

$$k_v{}^2 - \frac{\omega^2}{c^2} + \gamma^2 \left(\frac{1}{c^2} - \frac{1}{v'_\tau{}^2} \right) (\omega - \boldsymbol{v}_0 \boldsymbol{k}_v)^2 + \frac{\omega_p{}^2}{v'_\tau{}^2} = 0. \qquad \dots \quad (14b)$$

Eq. (14a) represents the dispersion relation for the electromagnetic mode whereas eq. (14b) represents the case for the plasma mode If the medium is not moving eq, (14) reduces to

$$k_c{}^2 - \frac{1}{c^2}(\omega^2 - \omega_p{}^2) = 0.$$

$$k_v{}^2 - (\omega^2 - \omega_p{}^2)/v_\tau{}^2 = 0$$

which are in good agreement with those of Cohen (1961). Using the condition $v_\tau << c$, eq. (14b) can be written as

$$k_v{}^2 - \frac{\gamma^2}{v'_\tau{}^2}(\omega - v_0 \boldsymbol{k}_u)^2 + \frac{\omega_p{}^2}{v'_\tau{}^2} = 0, \qquad \dots \quad (14b')$$

where the differential operator (11b) is written as

$$L_v = \nabla^2 - \frac{\gamma^2}{v'_\tau{}^2} \left(\frac{\partial}{\partial t} + v_0 \cdot \nabla \right)^2 - \frac{\omega_p{}^2}{v'_\tau{}^2} \qquad \dots \quad (11b')$$

Eq. (14b') represents the case when the compressibility of the medium is poor.

4. Summary

This paper has been concerned with the potential equations for the wave propagating through a moving compressible isotropic plasma. The dispersion relation for the electromagnetic as well as for the plasma mode propagation have been derived. Some particular cases have also been discussed.

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